Lesson 8.

Dynamic programming, cont.

Example 1. The Dijkstra Brewing Company is planning production of its new limited run beer, Primal Pilsner. The company must supply 1 batch next month, then 2 and 4 in successive months. Each month in which the company produces the beer requires a factory setup cost of $5,000. Each batch of beer costs $2,000 to produce. Batches can be held in inventory at a cost of $1,000 per batch per month. Capacity limitations allow a maximum of 3 batches to be produced during each month. In addition, the size of the company's warehouse restricts the ending inventory for each month to at most 3 batches. The company has no initial inventory.

The company wants to find a production plan that will meet all demands on time and minimizes its total production and holding costs over the next 3 months. Formulate this problem as a dynamic program by giving its shortest/longest path representation.

Example 2. The State of Maryland is conducting research on reducing traffic on the I-270 corridor. Three research teams are currently trying three different approaches to the problem. It has been estimated that the probability that the respective teams – call them 1, 2, and 3 – will fail is 0.40, 0.60, and 0.80, respectively.

In order to decrease probability of failure, the state wants to assign two additional researchers to the project. The following table gives the estimated probability that the respective teams will fail when 0, 1, or 2 additional researchers are added to that team:

	Probability of failure		
Number of new researchers	Team 1	Team 2	Team 3
0	0.40	0.60	0.80
1	0.20	0.40	0.50
2	0.15	0.20	0.30

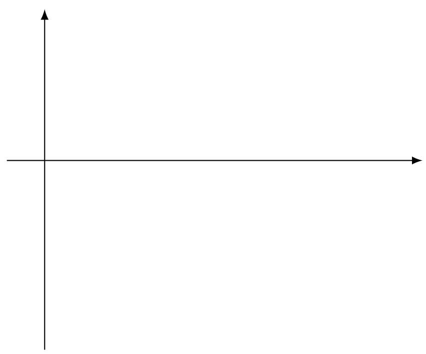
The state wants to determine how to allocate the two additional researchers in order to minimize the probability that all three teams will fail. Formulate this problem as a dynamic program by giving its shortest/longest path representation.

Products of costs and rewards

- Consider Example 2
- Let

$$p_i = \text{probability that team } i \text{ fails} \quad \text{for } i = 1, 2, 3$$

- We want to minimize the probability that all teams fail, or:
- We know how to find a shortest path – a path with the smallest sum of edge lengths
- What if wanted to find a path with the smallest product of edge lengths?
- Recall that the logarithm function is monotonic increasing:



- Therefore:
- So instead of setting the edge lengths equal to the probabilities, we can set the edge lengths equal to the logarithms of the probabilities
- Just make sure to invert the logarithms when interpreting the shortest path length

Example 3. To graduate from Simplexville University, Angie needs to pass at least one of the three courses she is taking this semester: literature, finance, and statistics. Angie's busy schedule of extracurricular activities allows her to spend only 4 hours per week on studying. Angie's probability of passing each course depends on the number of hours she spends studying for the course:

	Probability of passing course		
Hours of studying per week	Literature	Finance	Statistics
0	0.20	0.25	0.10
1	0.30	0.30	0.30
2	0.35	0.33	0.40
3	0.38	0.35	0.44
4	0.40	0.38	0.50

Angie wants to maximize the probability that she passes at least one of these three courses. Formulate this problem as a dynamic program by giving its shortest/longest path representation.

Hint. Why is maximizing the probability of passing at least one course equivalent to minimizing the probability of failing all three courses?